

Image Restoration Using Nonlocally Centralized Sparse Representation and histogram Clipping

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Abstract-

Due to the degradation of observed image the noisy, blurred, distorted image can be occurred. To restore the image information by conventional models may not be accurate enough for faithful reconstruction of the original image. I propose the sparse representations to improve the performance of based image restoration. In this method the sparse coding noise is added for image restoration, due to this image restoration the sparse coefficients of original image can be detected. The so-called nonlocally centralized sparse representation (NCSR) model is as simple as the standard sparse representation model, for denoising the image here we use the histogram clipping method by using histogram based sparse representation to effectively reduce the noise and also implement the TMR filter for Quality image. Various types of image restoration problems, including denoising, deblurring and super-resolution, validate the generality and state-of-the-art performance of the proposed algorithm.

Keywords —Image restoration, nonlocal similarity, sparse representation, histogram clipping.

I. Introduction

Image classification is one of the most important topics in computer vision. Recently, sparse coding technique attracts more and more attention because of its effectiveness in extracting global properties from signals. It recovers a sparse linear representation of a query datum with respect to a set of non-parametric basis set, known as dictionary [3, 24]. In image classification problem, methods based on sparse coding or its variants mainly collect a set of image patches to learn the dictionaries by representing an image with a histogram of local features, Bag of Words (BoW) models [25] have shown excellent performance, especially its robustness to spatial variations. Considering spatial information with BoW, Lazebnik et al. [10] built a spatial pyramid and extended the BoW model by partitioning the image into sub-regions and computing histograms of local features. Yang et al. [29] further extended Spatial Pyramid Matching (SPM) by using sparse coding. They provided a generalized vector quantization for sparse coding followed by multi-scale spatial max pooling.

Recent years have seen a great deal of renewed interests, enthusiasm and progress in sparsity-based image processing, particularly in image restoration. However, quite surprisingly, most published algorithms for image processing and analysis based themselves on the sparsity of luminance component of the image signal and overlooked the sparsities induced by spectral correlations. This leaves a slack in the performance of these algorithms. Mairalet. al. extended the K-SVD algorithm to color images in the searching of a dictionary based sparse representation of color images [2]. In this paper, to pick up the performance slack we investigate ways to formulate spectral correlations into inherent and computationally amenable sparse representations of multispectral images. Our investigation begins with an image formation model of digital color

Cameras. This image model and mild assumptions on illumination conditions and imaged objects reveal intrinsic sparsity properties of natural images. It turns out that these allow the newly revealed sparsities of color images to be readily exploited by a ℓ_1 minimization process, or by linear programming algorithmically. Upon the conclusion of our technical development, it will become self-evident how the new results of this paper can be integrated into the general framework of image restoration and used as strong domain knowledge to improve the solution of the corresponding inverse problem.

In this paper we improve the sparse representation performance by proposing a *nonlocally centralized sparse representation* (NCSR) model. To faithfully reconstruct the original image, the sparse code αy [refer to Eq. (3)] should be as close as possible to the sparse codes αx [refer to Eq. (2)] of the original image. In other words, the difference $\nu \alpha = \alpha y - \alpha x$ (called as sparse coding noise, SCN in short, in this work) should be reduced and hence the quality of reconstructed image $\hat{x} = \alpha y$ can be improved because $\hat{x} - x \approx \alpha y - \alpha x = \nu \alpha$. To reduce the SCN, we centralize the sparse codes to some good estimation of αx . In practice, a good estimation of αx can be obtained by exploiting the rich amount of nonlocal redundancies in the observed image

The proposed NCSR model can be solved effectively by conventional iterative shrinkage algorithm [9], which allows the remainder of the paper has the following flow of presentation. The image formation model is reviewed in Section II, which leads to the sparse representation that is detailed in Section III. Typical applications of color image denoising and deconvolution are investigated in Section IV. And finally, Section V concludes the paper.

II. NONLOCALLY CENTRALIZED SPARSE REPRESENTATION (NCSR)

Following the notation used in [19], for an image $x \in \mathbb{R}^{n \times n}$, let $x_i = \mathbf{R}_i x$ denote an image patch of size $n \times n$ extracted at location i , where \mathbf{R}_i is the matrix extracting patch x_i from x at location i . Given an dictionary $\mathbf{A} \in \mathbb{R}^{n \times M}$, $n \leq M$, each patch can be sparsely represented as $x_i \approx \mathbf{A} \alpha_i$, by solving an l_1 -minimization problem $\alpha_i = \arg \min_{\alpha} \{ \|\mathbf{A} \alpha - x_i\|_2 + \lambda \|\alpha\|_1 \}$. Then the entire image x can be represented by the set of sparse codes $\{\alpha_i, i\}$. The patches can be overlapped to suppress the boundary artifacts, and we obtain a redundant patch-based representation. Reconstructing x from $\{\alpha_i, i\}$ is an over-determined system, and a straightforward least-square solution is [19]: $x \approx (\sum_{i=1}^N \mathbf{R}_i^T \mathbf{R}_i)^{-1} \sum_{i=1}^N (\mathbf{R}_i^T x_i)$. For the convenience of expression, we let

$$\mathbf{r} = \mathbf{H} \mathbf{s} + \mathbf{n}, \quad (1)$$

Where \mathbf{r} denotes the concatenation of all α_i, i . The above equation is nothing but telling that the overall image is reconstructed by averaging each reconstructed patch of x_i . In the scenario of image restoration (IR), the observed image is modeled as $\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{v}$. The sparsity-based IR method recovers \mathbf{x} from \mathbf{y} by solving the following minimization problem:

$$\alpha_x = \arg \min_{\alpha} \{ \|\mathbf{y} - \mathbf{H} \alpha\|_2^2 + \lambda \|\alpha\|_1 \} \quad (2)$$

The image \mathbf{x} is then reconstructed as $\hat{\mathbf{x}} = \alpha_y$.

2.1. Sparse Coding Noise:

In order for an effective IR, the sparse codes α_y obtained by solving the objective function in Eq. (5) are expected to be as close as possible to the true sparse codes α_x of the original image \mathbf{x} . However, due to the degradation of the observed image \mathbf{y} (e.g., noisy and blurred), the sparse code α_y will deviate from α_x , and the IR quality depends on the level of the sparse coding noise (SCN), which is defined as the difference between α_y and α_x

$$\mathbf{v}_\alpha = \alpha_y - \alpha_x. \quad (3)$$

Image Lena as an example. In the first experiment, we add Gaussian white noise to the original image \mathbf{x} to get the noisy image \mathbf{y} (the noise level $\sigma_n = 15$). Then we compute α_x and α_y by solving Eq. (2) and Eq. (5), respectively. The DCT bases are adopted in the experiment. Then the SCN \mathbf{v}_α is computed. In Fig. 2(a-1), we plot the distribution of \mathbf{v}_α corresponding to the 4th atom in the dictionary. We also plot these distributions in log domain in Fig. 2(b-1) ~ (b-3). This observation motivates us to model \mathbf{v}_α with a Laplacian prior, as will be further discussed in Section III-A.

2.2. Modeling of NCSR:

The definition of SCN \mathbf{v}_α indicates that by suppressing the SCN \mathbf{v}_α we could improve the IR output $\hat{\mathbf{x}}$. However, the

difficulty lies in that the sparse coding vector α_x is unknown so that \mathbf{v}_α cannot be directly measured. Nonetheless, if we could have some reasonably good estimation of α_x , denoted by β , available, then $\alpha_y - \beta$ can be a good estimation of the SCN \mathbf{v}_α . To suppress \mathbf{v}_α and improve the accuracy of α_y and thus further improve the objective function of Eq., we can propose the following centralized sparse representation (CSR)

$$\alpha_y = \arg \min_{\alpha} \left\{ \|\mathbf{y} - \mathbf{H} \alpha\|_2^2 + \lambda \sum_i \|\alpha_i\|_1 + \gamma \sum_i \|\alpha_i - \beta_i\|_p \right\} \quad (4)$$

Where β_i is some good estimation of α_i , γ is the regularization parameter and p can be 1 or 2. In the above CSR model, while enforcing the sparsity of coding coefficients α_i , the sparse codes are also centralized to some estimate of α_x (i.e., β) so that SCN \mathbf{v}_α can be suppressed. One important issue of sparsity-based IR is the selection of dictionary \mathbf{A} .

Hence we propose the following sparse coding model:

$$\alpha_y = \arg \min_{\alpha} \left\{ \|\mathbf{y} - \mathbf{H} \alpha\|_2^2 + \lambda \sum_i \|\alpha_i - \beta_i\|_p \right\} \quad (5)$$

2.3. Nonlocal Estimate of Unknown Sparse Code:

Generally, there can be various ways to make an estimate of α_x , depending on how much the prior knowledge of α_x we have. If we have many training images that are similar to the original image \mathbf{x} , we could learn the estimate β of α_x from the training set. However, in many practical situations the training images are simply not available. On the other hand, the strong nonlocal correlation between the sparse coding coefficients, as shown in Fig. 1, allows us to learn the estimate β from the input data. Based on the fact that natural images often contain repetitive structures, i.e., the rich amount of nonlocal redundancies [30], we search the nonlocal similar patches to the given patch i in a large window centered at pixel i . For higher performance, the search of similar patches can also be carried out across different scales at the expense of higher computational complexity, as shown in [31]. Then a good estimation of α_i , i.e., β_i , can be computed as the weighted average of those sparse codes associated with the nonlocal similar patches (including patch i) to patch i . For each patch x_i , we have a set of its similar patches, denoted by i . Finally β_i can be computed from the sparse codes of the patches within i . Denote by $\alpha_{i,q}$ the sparse codes of patch $x_{i,q}$ within set i . Then β_i can be computed as the weighted average of $\alpha_{i,q}$

$$\beta_i = \sum_{q \in \Omega_i} \omega_{i,q} \alpha_{i,q} \quad (6)$$

Where $\omega_{i,q}$ is the weight. Similar to the nonlocal means approach [30], we set the weights to be inversely proportional to the distance between patches x_i and $x_{i,q}$

$$\omega_{i,q} = \frac{1}{W} \exp(-\|\hat{x}_i - \hat{x}_{i,q}\|_2^2 / h) \quad (7)$$

III. ALGORITHM OF NCSR

3.1. Parameters Determination:

In Eq. (8) or Eq. (11) the parameter λ that balances the fidelity term and the centralized sparsity term should be adaptively determined for better IR performance. In this subsection we provide a Bayesian interpretation of the NCSR model, which also provides us an explicit way to set the regularization parameter λ . In the literature of wavelet denoising, the connection between *Maximum a Posterior* (MAP) estimator and sparse representation has been established [28], and here we extend the connection from the local sparsity to nonlocally centralized sparsity. For the convenience of expression, let's define $\theta = \alpha - \beta$. For a given β , the MAP estimation of θ can be formulated as

$$\begin{aligned} \theta_y &= \arg \max_{\theta} \log P(\theta|y) \\ &= \arg \max_{\theta} \{\log P(y|\theta) + \log P(\theta)\}. \end{aligned} \quad (8)$$

The likelihood term is characterized by the Gaussian distribution where θ and β are assumed to be independent. In the prior probability $P(\theta)$, θ reflects the variation of α from its estimation β . If we take β as a very good estimation of the sparse coding coefficient of unknown true signal, then $\theta = \alpha - \beta$ is basically the SCN associated with αy , and we have seen in Fig. 2 that the SCN signal can be well characterized by the Laplacian distribution. Thus, we can assume that θ follows i.i.d. Laplacian distribution, and the joint prior distribution $P(\theta)$ can be modeled as

$$\begin{aligned} \theta_y &= \arg \min_{\theta} \left\{ \|y - H\Phi \circ \alpha\|_2^2 + 2\sqrt{2}\sigma_n^2 \right. \\ &\quad \left. \times \sum_i \sum_j \frac{1}{\sigma_{i,j}} |\theta_i(j)| \right\}. \end{aligned} \quad (9)$$

Hence, for a given β the sparse codes α can then be obtained by minimizing the following objective function Compared with Eq. (8), we can see that the l_1 -norm (i.e. $p=1$) should be chosen to characterize the SCN term $\alpha i - \beta i$. Comparing Eq. (16) with Eq. (8), we have

$$\lambda_{i,j} = \frac{2\sqrt{2}\sigma_n^2}{\sigma_{i,j}}. \quad (10)$$

3.3. Histogram-Based Sparse Representation:

Histogram-based representations have been widely used with the feature descriptors, e.g., HOG [4], BoW [25], and GLOH [17]. It provides very compact representation and captures global frequency of low-level features. In this section, we present a framework that determines the component-level importance of histogram information and Combines it with a sparse representation, which is referred to as Histogram-Based Component-Level Sparse Representation (HCLSP) for the rest of this paper.

3.3.1. Component Level Importance:

Suppose we have a training set of image groups. Each image group is defined as a class. For the training set, we have C classes, for each class index $p = 1; \dots; C$. Denote $X(p) = [x(p)_1; \dots; x(p)_n]$ in $R^{m \times n}$ to be a set of training samples from class p , with each individual sample $x(p)_i$ in R^m . The dictionaries trained from class p are represented by $D(p)$. Given the training data and the

dictionaries, the sparse coefficient vector α can be obtained by solving equation (2) using LARS-Lasso or other standard algorithms. Denote the reconstruction error for the training set $X(p)$ by using the dictionaries $D(p)$ as:

$$R^{(p)}(X^{(p)}) = \sum_{i=1}^n |x_i^{(p)} - D^{(p)} \alpha_i^{(p)}| \quad (11)3.4.$$

Histogram Equalization:

Histogram equalization is a technique for adjusting image intensities to enhance contrast. Let f be a given image represented by $m \times n$ matrix of integer pixel intensities ranging from 0 to $L - 1$. L is the number of possible intensity values.

IV. EXPERIMENTAL RESULTS

To verify the IR performance of the proposed NCSR algorithm we conduct extensive experiments on image denoising, deblurring and super-resolution. The basic parameter setting of NCSR is as follows: the patch size is 7×7 and $K = 70$. For image denoising, $\delta = 0.02$, $L = 3$, and $J = 3$; for image deblurring and super-resolution, $\delta = 2.4$, $L = 5$, and $J = 160$. To evaluate the quality of the restored images, the PSNR and the recently proposed powerful perceptual quality metric FSIM [32] are calculated. Due to the limited page space, we only show part of the results in this paper, and all the experimental results can be downloaded on the clearer and much more details are recovered. Considering that the estimated kernel will have bias from the true unknown blurring kernel, these experiments validate that NCSR is robust to the kernel estimation errors.

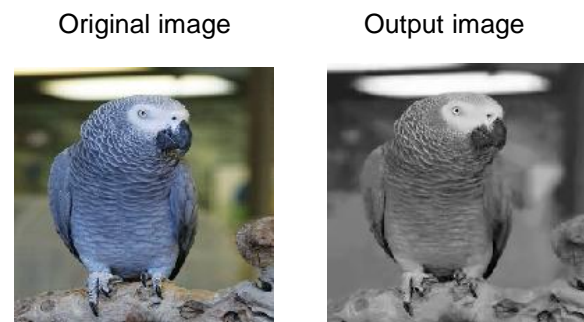


Fig .1 conversion of image to gray scale image

We also test the proposed NCSR deblurring method on real motion blurred images. Since the blur kernel estimation is a non-trivial task, we borrowed the kernel estimation method from [34] to estimate the blur kernel and apply the estimated blur kernel in NCSR to restore the original images. In Fig. 8 we present the deblurring results by the blind deblurring method of [34] and the proposed NCSR method. We can see that the images restored by our approach are much clearer and much more details are recovered. Considering that the estimated kernel will have bias from the true unknown blurring kernel, these experiments validate that NCSR is robust to the kernel estimation errors



Original Image Histogram Spatial adaptive
 equalization Histogram
 equalization



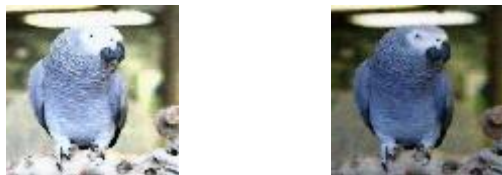
Spatial preserving equalization Histogram clipping

Fig.2 image denoising outout image with the quality



Zoom before Iterated TMR Zoom after Iterated
 TMR

Fig.3 TMR filtered Image



Iterated TMR Filter of Image (b) Iterated TMR
 Filter of (c)



Iterated TMR Filter of Image iterated TMR filter
 of (d) of image (e)

Fig.4 Denoising image

V. CONCLUSION

In this paper we presented a novel nonlocally centralized sparse representation (NCSR) model for image restoration. The sparse coding noise (SCN), which is defined as the difference between the sparse code of the degraded image and the sparse code of the unknown original image, should be minimized to improve the performance of sparsity-based image restoration. To this end, we proposed a centralized sparse constraint, which exploits the image nonlocal redundancy, to reduce the SCN. The Bayesian interpretation of the NCSR model was provided and this endows the NCSR model an iteratively reweighted implementation. An efficient iterative shrinkage function was presented for solving the l_1 -regularized

NCSR minimization problem. Experimental results on image denoising, deblurring and super-resolution demonstrated that the NCSR approach can achieve highly competitive performance to other leading denoising methods, and outperform much other leading image deblurring and super-resolution methods. And also implement the histogram clipping and TMR filter for Quality of picture.

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